Data Station A Recording Sheet

Data Source	http://exploringdata.cqu.edu.au/datasets/oil_prod.xls
How would you describe this set of data? Why?	
What relationships are found within this set of data? Why?	
How would you represent this data? Why?	
What question(s) can we pose to students that this set of data helps to answer?	
How might this data be used to extend what students already understand about our course content?	

Data Station B Recording Sheet

Data Source	*Time Almanac 2005*, "Coastline of the United States," page 502.
How would you describe this set of data? Why?	
What relationships are found within this set of data? Why?	
How would you represent this data? Why?	
What question(s) can we pose to students that this set of data helps to answer?	
How might this data be used to extend what students already understand about our course content?	

Data Station C Recording Sheet

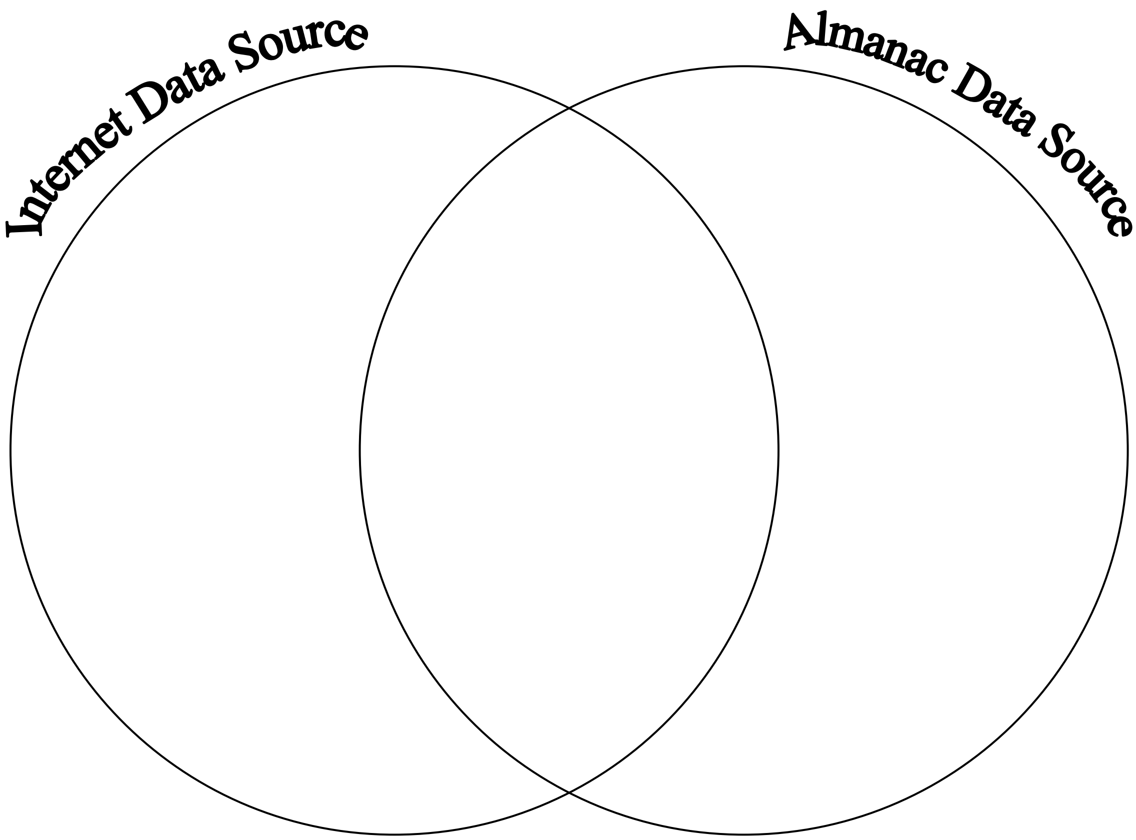
Data Source	CBR, graphing calculator, different sized beach balls
What set of data can you generate with these tools?	
What relationships are found within this set of data? Why?	
How would you represent this data? Why?	
What question(s) can we pose to students that this set of data helps to answer?	
How might this data be used to extend what students already understand about our course content?	

Data Station D Recording Sheet

Data Source	One-inch cubes, yard sticks
What set of data can you generate with these tools?	
What relationships are found within this set of data? Why?	
How would you represent this data? Why?	
What question(s) can we pose to students that this set of data helps to answer?	
How might this data be used to extend what students already understand about our course content?	

Reflections on Data

Complete the following Venn Diagram to compare and contrast the uses of the internet and an almanac as data sources.



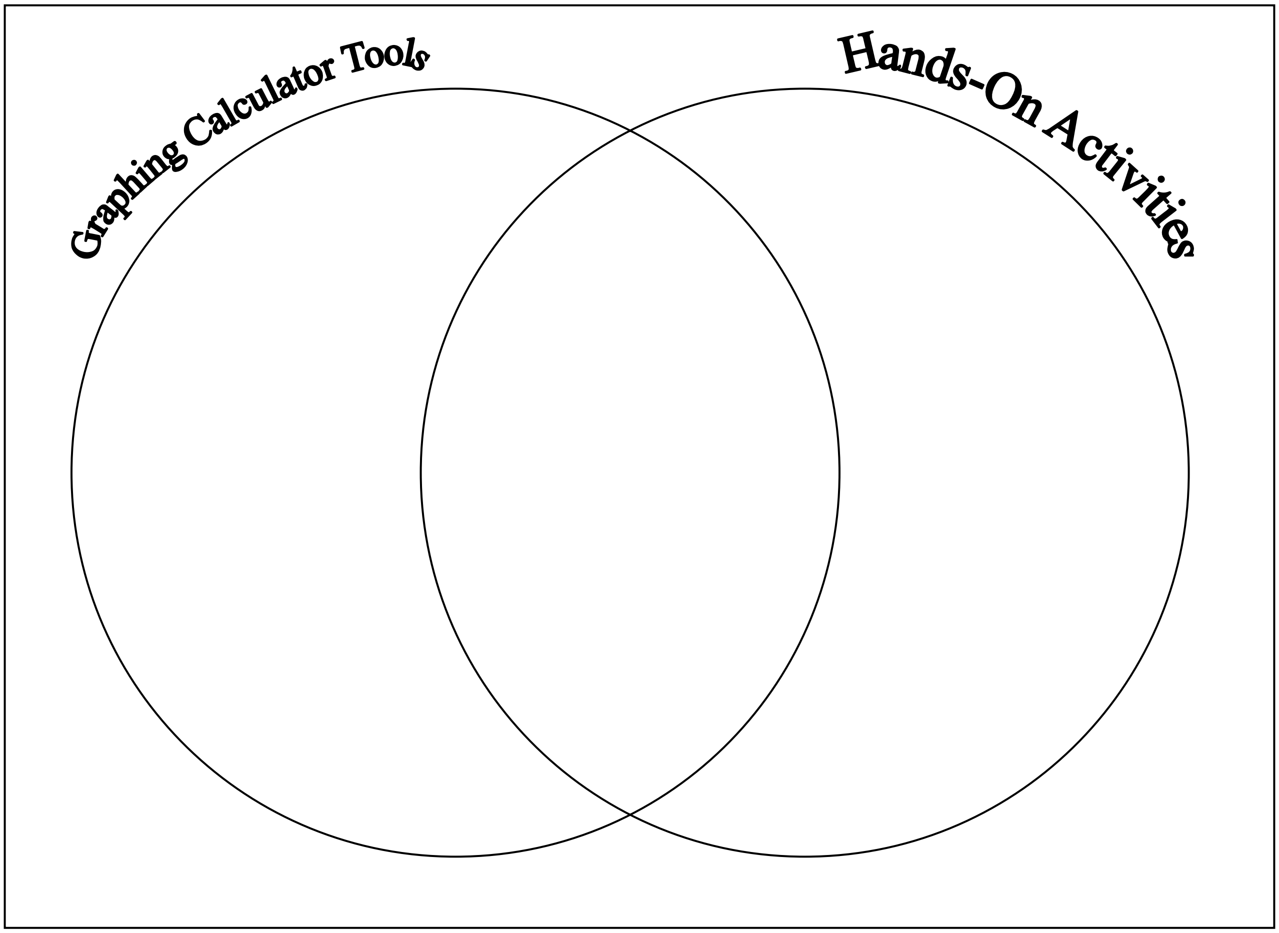
What are the benefits of using data found on the Internet?

What are the benefits of using data found in print sources such as an almanac?

How might teachers use these data sources in an Algebra 2 classroom?

Reflections on Data

Complete the following Venn Diagram to compare and contrast the uses of the graphing calculator tools and hands-on activities as data sources.



What are the benefits of using data resulting from graphing calculator tools?

What are the benefits of using data derived from hands-on activities?

How might teachers use these data sources in an Algebra 2 classroom?

Debriefing the Exploration of Data

1. What questions can we ask as reflective practitioners to determine the appropriateness of a data source for promoting mathematical learning?

2. How does the technology-based data offer an opportunity to strengthen mathematical learning?

3. How might hands-on activities complement the judicious use of technology?

4. What paper-and-pencil methods do students need to know to make sense of the data we explored?

Planning for Intentional Use of Data in the Classroom

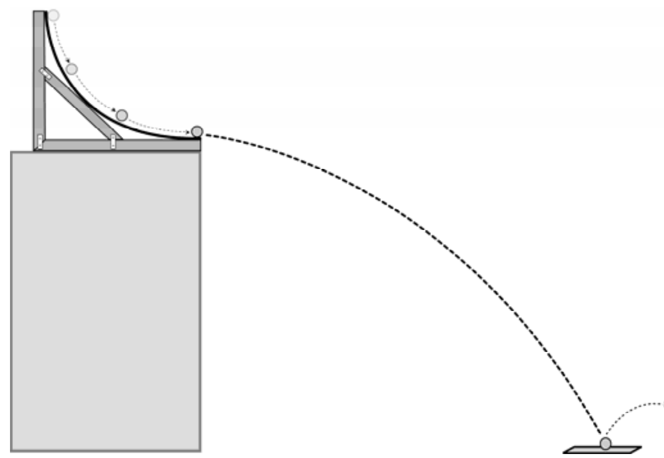
TEKS			
Question(s) to Pose to Students	Math		
	Tech		
Cognitive Rigor	Knowledge		
	Understanding		
	Application		
	Analysis		
	Evaluation		
	Creation		
Data Source(s)	Real-Time		
	Archival		
	Categorical		
	Numerical		
Setting	Computer Lab		
	Mini-Lab		
	One Computer		
	Graphing Calculator		
	Measurement-Based Data Collection		
Bridge to the Classroom			

Flying Off the Handle

In Hollywood movies, a classic way to end a car chase is to have a car drive off a cliff or into a canyon, with the hero jumping out of the car at the last minute to safety. Movie directors need to know exactly where the car will land so that they can have cameras in place to capture the motion on film. They also need to know where the car will be as it falls from the edge of the cliff to the floor of the canyon below so that they can have cameras in place there, too. How can we model this motion? How can we harness technology to apply this model to pinpoint the specific location of a moving object at any time?

To answer these questions, let's build a model that will enable us to simulate a car driving off a cliff. Use a wooden ramp and marble to simulate the car's motion. How would we develop a function rule to determine the placement of the cup on top of a stack of textbooks?

Set up the ramp on a high table. Roll the marble down the ramp and let it hit the floor to observe the motion of the marble.



1. Let the floor represent the *x*-axis and the end of the ramp be contained on the *y*-axis. Where would the origin of this coordinate system be?

2. In this coordinate system, what do *x* and *y* represent?

3. Consider the path of the marble. Based on your coordinate system, what does the *y*-intercept represent? What are the coordinates of the *y*-intercept? Record the coordinates as a point in the table.

Horizontal Distance (*x*)	Height of the Marble (*y*)

4. Based on your coordinate system, what does the *x*-intercept represent?

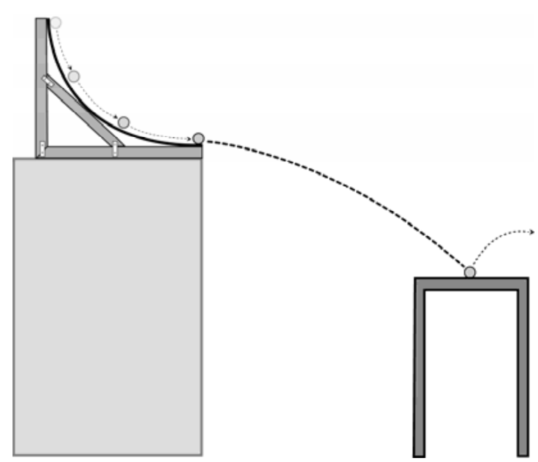
Roll the marble down the ramp and notice where it strikes the floor. Tape a piece of carbon paper on top of a piece of typing paper (carbon side down) where the marble touched the floor. Tape the paper to the floor.

Roll the marble down the ramp and let it strike the paper on the floor. Repeat at least twice so that you have data for at least 3 trials.

5. What are the coordinates of the *x*-intercept? Record the coordinates as a point in the table.

6. Place a chair or desk between the ramp and the point of impact on the floor. Repeat your data collection procedure to find the *x*- and *y*-coordinates of the point of impact on the chair or desk. Record your third data point in the table.

7. What kind of functional relationship do you think exists between the horizontal distance and the vertical distance of the marble?



8. Make a scatterplot of your data. Sketch your plot.

9. Use the coordinates of the three data points to write a function rule that could be used to predict the height of the marble, *y*, when it is a horizontal distance, *x*, from the ramp. Explain how you found your function.

10. Graph your function rule over your scatterplot and sketch your graph. Is the function rule a good fit? How can you tell? If not, how can you revise your function rule so that it is a better fit?

11. Place your cup on top of three textbooks. Where do you need to place the cup so that the marble will roll off the ramp and land inside the cup? Justify your choice.

12. Test your prediction. Was your prediction correct? Why or why not? If not, revise your prediction and test it again.

Flying Off the Handle: Intentional Use of Data

TEKS			
Question(s) to Pose to Students	Math		
	Tech		
Cognitive Rigor	Knowledge		
	Understanding		
	Application		
	Analysis		
	Evaluation		
	Creation		
Data Source(s)	Real-Time		
	Archival		
	Categorical		
	Numerical		
Setting	Computer Lab		
	Mini-Lab		
	One Computer		
	Graphing Calculator		
	Measurement-Based Data Collection		
Bridge to the Classroom			

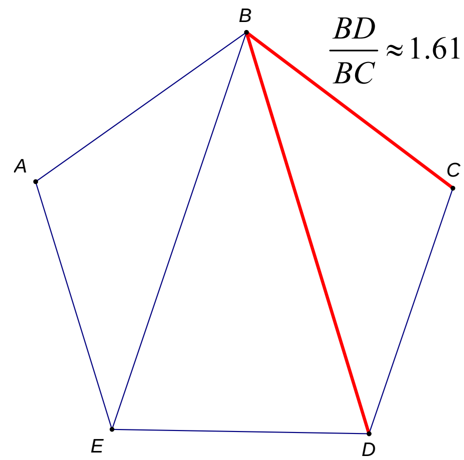
A Golden Idea

Mathematics has been an important influence throughout civilization, from ancient times to the present. In ancient Egypt, Greece, and Rome, geometry and proportion were used in art and architecture. Medieval Europeans carried this tradition of using proportion as they built beautiful cathedrals. Renaissance painters and sculptors used proportion to convey their idea of natural beauty.

Where does this proportion originate? The Greeks found a certain ratio to be prevalent in the natural world around them.

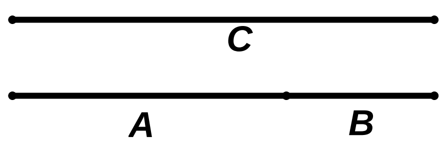
The golden ratio can be developed from several constructions. One way to construct the golden ratio is using the diagonals of a regular pentagon.

In regular pentagon *ABCDE* (shown at right), the ratio of the length of a diagonal from vertex B to the length of a side of the pentagon is always the same. This ratio is called the **golden ratio**, which the Greeks notated with the capital letter *phi*, or Φ.



$$\frac{BD}{BC} \approx 1.618 = \Phi$$

From a segment length perspective, the golden ratio is a geometric mean. Geometrically, segment (in the diagram below, of length *C*) can be split into two smaller segments (of lengths *A* and *B*).



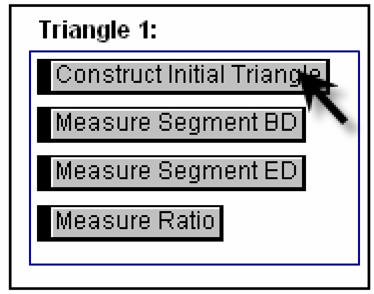
The splitting of the segment is such that the ratio of the length of the original segment to the length of the larger piece (*C*:*A*) is the same as the ratio of the length of the larger piece to the length of the smaller piece (*A*:*B*). In other words,

$$\frac{C}{A} = \frac{A}{B}$$

If the golden ratio is applied in succession to a geometric construction, what types of functional behaviors are present?

Part 1: Investigating Leg Length

Open the Geometer's Sketchpad sketch "Golden Triangles.GSP." Pentagon *ABCDE* is a regular pentagon. From this regular pentagon, a series of triangles can be constructed. Click on the "Construct Initial Triangle" action button.

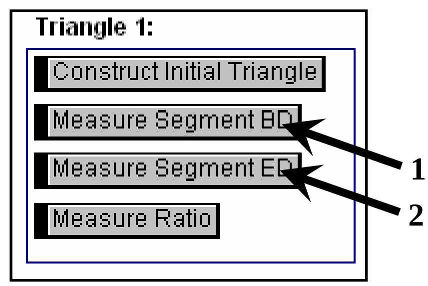


1. What kind of triangle is ΔBED? How do you know?

2. Is your triangle classification from Question 1 true for every triangle formed when two diagonals are drawn from one vertex of a regular pentagon? How do you know?

Measure the length of $\overline{BD}$ by clicking on the "Measure Segment BD" action button. Measure the length of $\overline{ED}$ by clicking on the "Measure Segment ED" action button.

3. What is the ratio of the length of $\overline{BD}$ to the length of $\overline{ED}$? How did you find this ratio?



4. What does this ratio represent?

Click on the "Construct Triangle 1" button. This animation bisects angle *BED*, then rotates the resulting triangle 108° to the same orientation as the original triangle. Measure the length of $\overline{CG}$ by clicking on the "Measure Segment CG" button.

5. What is the ratio of $\frac{BD}{CG}$? $\frac{CG}{BD}$? How do these numbers compare?

6. How does ΔCDG compare to ΔBED? How do you know?

7. What scale factor could be applied to ΔBED to generate ΔCDG? Have you seen this ratio before? If so, where?

Click on the “Construct Triangle 2” button. This animation constructs ΔJGK in the same manner as the construction of ΔCDG. Measure the length of $\overline{JK}$ by clicking on the “Measure Segment JK” button.

8. How does ΔJGK compare to ΔCDG? How do you know?

Click the “Construct Triangle 3” button. This animation constructs ΔMKN in the same manner as the construction of ΔJGK. Measure the length of $\overline{MN}$ by clicking the “Measure Segment MN” button.

9. How does ΔMKN compare to ΔJGK? How do you know?

Click the “Construct Triangle 4” button. This animation constructs $\Delta \boldsymbol{QNR}$ in the same manner as the construction of $\Delta \boldsymbol{MKN}$. Measure the length of $\overline{\boldsymbol{QR}}$ by clicking the “Measure Segment QR” button.

10. How does $\Delta \boldsymbol{QNR}$ compare to $\Delta \boldsymbol{MKN}$? How do you know?

11. What patterns do you observe in the sequence of triangles?

12. Record the measures of the leg of each triangle in the following table.

Triangle		**Name of Leg**	**Length of Leg**	**Process**	**Ratio**
Name	**#**				
ΔBED					
ΔCDG					$\frac{CG}{BD} =$
ΔJGK					$\frac{JK}{CG} =$
ΔMKN					$\frac{MN}{JK} =$
ΔQNR					$\frac{QR}{MN} =$

13. Record the ratio of each leg length to its previous leg length in the table.

14. Use an appropriate technology to generate a scatterplot of Leg Length vs. Triangle Number (let $\Delta \boldsymbol{BED}$ be Triangle Number 0). Sketch your scatterplot and indicate the dimensions of the values on your *x*-axis and *y*-axis.

15. Based on your scatterplot, what type of function would model the relationship found in the data? Justify your choice.

16. Use the parent function from Question 15 to determine a function rule to describe the relationship between triangle number and leg length. What do the variables in your function rule represent? What do the constants in your function rule represent?

17. Graph your function rule over the scatterplot. Sketch your graph. How well does the function rule describe your data?

18. Compare the domain of your data and the domain of the function rule.

19. Compare the range of your data and the range of the function rule.

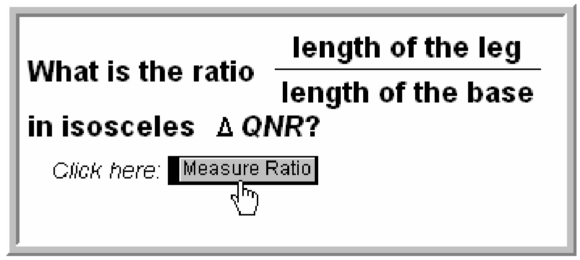
20. What will be the length of the leg of the 9th triangle in this sequence? Explain how you determined your answer.

21. Which triangle will be the first one to have a leg length less than 0.5 cm? Explain how you determined your answer.

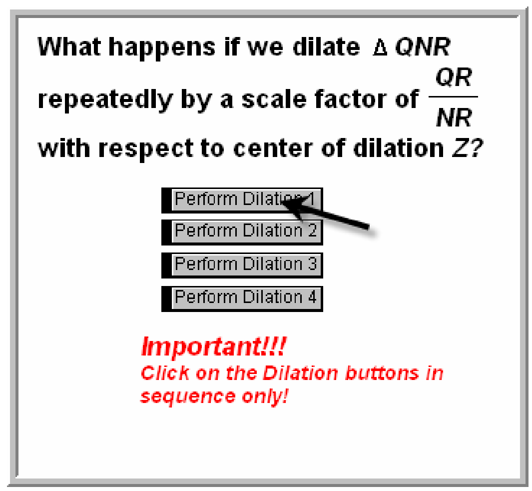
Part 2: Investigating Dilations

In the previous activity, you constructed a series of golden isosceles triangles. What happens if we take a golden triangle and enlarge it repeatedly?

1. In the same Geometer's Sketchpad sketch, click on the "Investigating Dilations" tab. In isosceles ΔQNR, what is the ratio of the length of the leg, QR, to the length of the base, NR?



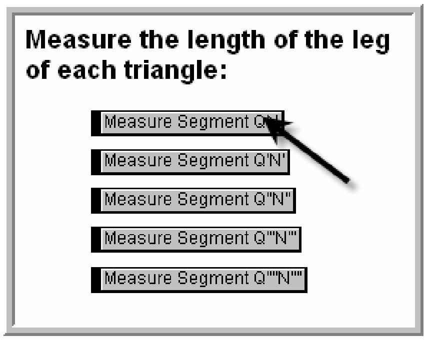
2. Click the "Perform Dilation 1 button." Describe what you see.



3. Click the remaining "Perform Dilation" buttons in sequential order, one at a time. Describe the result.

4. How do each of the triangles compare with each other? How do you know?

5. Measure the leg lengths of each triangle by clicking the "Measure Segment" buttons in order, one at a time. Record the segment lengths in the table below.



Triangle		Name of Leg	Length of Leg	Process	Ratio
Name	**Dilation Number**				
ΔQNR	*0*				
$\Delta Q'N'N$	*1*				$\frac{Q'N'}{QN} =$
$\Delta Q''N''N'$	*2*				$\frac{Q''N''}{Q'N'} =$
$\Delta Q'''N'''N''$	*3*				$\frac{Q'''N'''}{Q''N''} =$
$\Delta Q''''N''''N'''$	*4*				$\frac{Q''''N''''}{Q'''N'''} =$

6. Record the successive ratios in the appropriate column of your table. Use an appropriate technology to generate a scatterplot of Leg Length vs. Dilation Number. Sketch your scatterplot and indicate the dimensions of the values on your *x*-axis and *y*-axis.

7. Based on your scatterplot, what type of function would model the relationship found in the data? Justify your choice.

8. Use the parent function from Question 7 to determine a function rule to describe the relationship between dilation number and leg length. What do the variables in your function rule represent? What do the constants in your function rule represent?

9. Graph your function rule over the scatterplot. Sketch your graph. How well does the function rule describe your data?

10. What scale factor could be used to generate the second dilation from the original triangle without generating the first dilation?

11. What scale factor could be used to generate the third dilation from the original triangle without generating the first two dilations?

12. How could you predict the scale factor in terms of the dilation number?

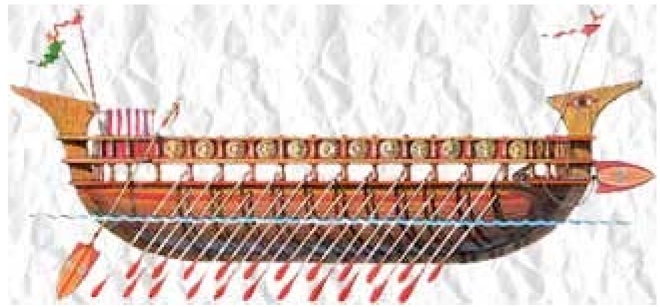
13. What scale factor would be used to generate the 9th dilation in the sequence? What would the leg length of this triangle be? Explain how you determined your answer.

14. Which dilation will be the first one to have a leg length of at least 2.5 meters? Explain how you determined your answer.

A Golden Idea: Intentional Use of Data

TEKS		
Question(s) to Pose to Students	**Math**	
	Tech	
Cognitive Rigor	Knowledge	
	Understanding	
	Application	
	Analysis	
	Evaluation	
	Creation	
Data Source(s)	Real-Time	
	Archival	
	Categorical	
	Numerical	
Setting	Computer Lab	
	Mini-Lab	
	One Computer	
	Graphing Calculator	
	Measurement-Based Data Collection	
Bridge to the Classroom		

I've Seen the Light!



A Phoenician boat captain was enjoying the cool Mediterranean breeze as his boat sailed from Tyre to Carthage with another shipment of purple dye for the king. The 1200-mile voyage was not easy. The captain smiled to himself as he thought of the many boats from other nations that became lost at sea attempting to make this journey. He and his Phoenician counterparts had a leg up on the competition- they knew how to use the stars to navigate. The captain looked up at the night sky, noticing the stars to make sure he was on course. He was always amazed by the variety of stars, some blue and white, some yellow and red. Some bright, some faint.

We know today that the brightness of a star depends on several factors. One factor is the distance between the Earth and the star. These distances are difficult to measure, so they must be calculated. In order to calculate these distances, astronomers must first know the relationship between the intensity of starlight and the distance between the Earth and the star.

To solve this problem, we can use the problem-solving strategy of "solving a simpler problem." To do so, use a flashlight to simulate a star and use a light-intensity sensor to measure the intensity of the light for varying distances.

Attach a light sensor to your data collection device and graphing calculator. Run a program, such as the DataMate APP, that measures intensity of light to collect data. One person in the group should hold the light sensor as another person walks towards the sensor with the flashlight.

1. Use the light sensor to collect data in intervals of 0.1 meter. See ***Technology Tutorial: Using the CBL2 to Collect Light Data*** for detailed instructions if necessary. Record your data in the table.

Distance (*D*) (m)	**Intensity (*I*) (mW/cm^2)**	**Distance (*D*) (m)**	**Intensity (*I*) (mW/cm^2)**

2. Using an appropriate technology, generate a scatterplot of your data. Sketch your scatterplot.

3. Find an appropriate function rule to model your data. Test the rule over your scatterplot. Sketch your graph.

4. A plant will be placed 275 centimeters from the light source. What intensity of light will it receive? Justify your answer.

5. A particular solar cell needs to receive at least 0.4 milliwatts per square centimeter of light to generate enough electricity to power a small toy. How far from the light source should the solar cell be placed in order to begin powering the toy? Justify your answer.

I've Seen the Light!: Intentional Use of Data

TEKS		
TEKS		
Question(s) to Pose to Students — Math		
Question(s) to Pose to Students — Tech		
Cognitive Rigor	Knowledge	
	Understanding	
	Application	
	Analysis	
	Evaluation	
	Creation	
Data Source(s)	Real-Time	
	Archival	
	Categorical	
	Numerical	
Setting	Computer Lab	
	Mini-Lab	
	One Computer	
	Graphing Calculator	
	Measurement-Based Data Collection	
Bridge to the Classroom		

The Doomsday Model

In 1960, Heinz von Foerster, Patricia Mora, and Larry Amiot, three scientists from the University of Illinois, published "Doomsday: Friday, 13 November, AD 2026" in the journal *Science*. In their paper, they considered the past population growth of the world and the current state (as of 1960) of the world's resources and their ability to sustain a certain population. They developed a model to describe population growth. A simplified variation of this model, where t represents the year and P represents the world population in billions, is:

$$P = \frac{195}{2026 - t}$$

They used this rational function to decide when the world's population would reach an unsustainable level and called this date "doomsday."

Use the Internet to obtain world population data since 1960. How well did the Doomsday Model describe the world's population growth between 1960 and 2000? How well does the model describe the world's population today? Based on the population data you found, how would you revise the model? When does this model predict "doomsday" will occur?

Share your results and your revised model with the group.

Gallery Walk Observations

Explore/Explain I: Flying Off the Handle	How did the activity promote careful decision making about the use of technology? How did the activity integrate technology into the learning of mathematics? Was technology use ever restricted for the purpose of enhancing learning? Why? How did the technology facilitate discussion about “algebraic sense”?

Explore/Explain II: A Golden Idea	How did the activity promote careful decision making about the use of technology? How did the activity integrate technology into the learning of mathematics? Was technology use ever restricted for the purpose of enhancing learning? Why? How did the technology facilitate discussion about “statistical sense”, “algebraic sense”, or “geometric sense”?

Explore/Explain III: I've Seen the Light!	How did the activity promote careful decision making about the use of technology? How did the activity integrate technology into the learning of mathematics? Was technology use ever restricted for the purpose of enhancing learning? Why? How did the technology facilitate discussion about "statistical sense" or "algebraic sense"?

Elaborate: The Doomsday Model	How did the activity promote careful decision making about the use of technology? How did the activity integrate technology into the learning of mathematics? Was technology use ever restricted for the purpose of enhancing learning? Why? How did the technology facilitate discussion about "statistical sense" or "algebraic sense"?